METAMATERIAL THEORY FOR LOW TEMPERATURE HEAT CAPACITY OF PHONONIC CRYSTAL MEMBRANES

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Phononic crystal (PnC) membranes have been shown to possess unintuitive thermal properties at sub-Kelvin temperature region. For instance, holey periodic patterning in a thin membrane of insulating material can be designed to reduce thermal conductance by more than an order of magnitude when compared to similar full membranes without patterning. [1] In contrast, the thermal conductance and the specific heat capacity of PnCs can also be increased by removing matter. [2,3] These effects are caused by phonon modes whose dispersion relations are heavily modified from the corresponding full membrane Rayleigh-Lamb modes.

For a typical PnC membrane, the calculation of the low temperature heat capacity requires a numeric evaluation of the dispersion relations over densely sampled space of wave vectors, which can be computationally expensive. Trying to overcome this issue, we have recently showed that simple 2D or 3D Debye models for specific heat capacity are not adequate at the low temperature limit. [3] In the present work we derive a new "Debye-like" theory for heat capacity, which is accurate in the low-temperature, thermal metamaterial limit, where the mechanical properties of the PnC membranes are described by the use of effective medium theories of elasticity. Moreover, in the zero hole filling fraction limit, Lamb-mode results obtained in [4] are recovered.

In the effective medium approximation, the PnC membrane is modeled as a full membrane made of an imaginary anisotropic material whose overall mechanical response is described by an effective elasticity tensor C*, which can be presented semi-analytically in closed form. [5] By calculating the low frequency asymptotes for the first three phonon modes, we derive a formula for the specific heat capacity of a PnC membrane at the low temperature limit. We also discuss geometric and physical limitations of the theory, and use it to predict the low temperature heat capacity of PnC membranes of different filling factors and materials.

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